

On localization of universal scalar fields in a tachyonic de Sitter thick braneworld

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Braneworld models may yield interesting effects ranging from high-energy physics to cosmology, or even some low-energy physics. Their mode structure modifies standard results in these physical realms that can be tested and used, for example, to set bounds on the models parameters. Now, to define braneworld deviations from standard 4D physics, a notion of matter and gravity localization on the brane is crucial. In this work we investigate the localization of universal massive scalar fields in a de Sitter thick tachyonic braneworld generated by gravity coupled to a tachyonic bulk scalar field. This braneworld possesses a 4D de Sitter induced metric and is asymptotically flat despite the presence of a negative bulk cosmological constant, a novel and interesting peculiarity that contrasts with previously known models. It turns out that universal scalar fields can be localized in this expanding braneworld if their bulk mass obeys an upper bound, otherwise the scalar fields delocalize: The dynamics of the scalar field is governed by a Schrödinger equation with an analog quantum mechanical potential of modified Pöschl-Teller type. This potential depends on the bulk curvature of the braneworld system as well as on the value of the bulk scalar field mass. For masses satisfying a certain upper bound, the potential displays a negative minimum and possesses a single massless bound state separated from the Kaluza-Klein (KK) massive modes by a mass gap defined by the Hubble (expansion scale) parameter of the 3-brane. As the bulk scalar field mass increases, the minimum of the quantum mechanical potential approaches a null value and, when the bulk mass reaches certain upper bound, it becomes positive (eventually transforming into a potential barrier), leading to delocalization of the bulk scalar field from the brane. We present analytical expressions for the general solution of the Schrödinger equation. Thus, the KK massive modes are given in terms of general Heun functions as well as the expression for the massless zero mode, giving rise to a new application of these special functions.

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I. INTRODUCTION AND REVIEW OF THE MODEL

The study of braneworlds has considerably evolved in the last years. Within the braneworld paradigm theoretical science has found consistent alternatives to solve or reformulate various problems of modern physics, such as the cosmological constant problem [1]–[2], the gauge hierarchy problem [3]–[7] or possible low energy effects including the Lamb shift in Hydrogen [8], the Casimir force [9] and non-singular particle potentials [10], among others (for an interesting review see [11]). In this context, there are many models of thin branes [5, 6] and thick branes [12] which shape the fifth dimension using different matter fields living in the bulk. The freedom we have to use matter fields in the bulk has led some authors to use all kinds of bulk fields, scalar fields being the most popular among them [11]–[27].

In order to model the fifth dimension, the use of a 5D tachyonic scalar field was explored in [20, 27], however along this research line, some attempts failed to localize 4D gravity (and other matter fields) or to find real solutions to the system of highly nonlinear equations. In fact, the tachyon condensate proposed by A. Sen [28] presents an additional difficulty due to the complex form of the field equations, however, despite these difficulties, in a recent work [27], an

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interesting and novel solution which localizes 4D gravity using a real tachyon field was obtained. Subsequently, the localization of gauge vector fields in this braneworld was recently accomplished and reported in the literature [29] (see [30] as well for an alternative mechanism for localizing gauge and Kalb–Ramond fields on thick branes based on a 5D Stueckelberg action, and [31]–[32] for its canonical quantization). A peculiar feature of this expanding thick braneworld model is that both the gravitational and gauge vector fields only possess a *single* massless bound state localized on the 3-brane in their corresponding mass spectra, allowing us to successfully recover the phenomenology of our 4D world. This expanding de Sitter braneworld is also interesting from the cosmological point of view since it qualitatively describes both the inflationary period of the early Universe and the accelerating expansion that we observe nowadays. In particular, since the cosmic inflation theory is in quite good agreement with the properties of the temperature fluctuations observed in the Cosmic Microwave Background Radiation, and since inflation likely took place at very high temperatures and, hence, its study involves some assumptions related to the relevant physics that takes place at such high energies, cosmologists have performed several attempts to construct consistent inflationary models within string theory and supergravity [33–35].

In this work we will continue with the study of localization of universal massive scalar fields (for localization of matter see [36]–[40]), on the aforementioned tachyonic thick braneworld [27], following [29], where the possibility of localizing bulk gauge vector fields was successfully explored. This paper is organized in three parts. In the first section we present an introduction and a brief review of the derivation of the de Sitter tachyonic thick braneworld, together with some of its relevant physical properties. In the second section we study the localization of massive universal scalar fields, and we find that the massless zero mode is localized on the brane only in the case when its 5D mass obeys an upper bound, otherwise delocalizing from it; on the other hand, in the mass spectrum there is also a tower of Kaluza–Klein (KK) delocalized massive modes separated from the massless one by a mass gap. The corresponding expressions for both the massless and KK massive modes are given in terms of general Heun functions. Finally, in the third section we present our conclusions and discuss a little bit on the obtained results.

A. Review of the tachyonic de Sitter thick braneworld

Let us start with the following 5D action for a thick braneworld model that emerges from the interaction of gravity with a bulk tachyonic scalar field in the presence of a bulk cosmological constant [27]

$$S = \int d^5x \sqrt{-g} \left(\frac{1}{2\kappa_5^2} R - \Lambda_5 - V(T) \sqrt{1 + g^{AB} \partial_A T \partial_B T} \right), \quad (1)$$

where R is the 5D scalar curvature, Λ_5 is the bulk cosmological constant, and $\kappa_5^2 = 8\pi G_5$ with G_5 the five-dimensional Newton constant. Here we use the signature $(- + + +)$ and the Riemann tensor, defined as follows $R_{MNPQ} = \frac{\Lambda_5}{6} (g_{MP}g_{NQ} - g_{MQ}g_{NP})$, gives rise to the Ricci one $R_{NQ} = R_{NM}^M$ upon contraction of its first and third indices, where $M, N, P, Q = 0, 1, 2, 3, 5$. From this action, the Einstein equations with a cosmological constant in five dimensions are

$$G_{AB} = -\kappa_5^2 \Lambda_5 g_{AB} + \kappa_5^2 T_{AB}^{bulk} \quad (2)$$

where T_{AB}^{bulk} is the energy–momentum tensor for the bulk tachyonic scalar field.

A 5D metric ansatz compatible with an induced 3-brane with spatially flat cosmological background can be taken to be

$$ds^2 = e^{2f(w)} [-dt^2 + a^2(t) (dx^2 + dy^2 + dz^2) + dw^2], \quad (3)$$

where $e^{2f(w)}$ and $a(t)$ are the warp factor and the scale factor of the brane and w stands for the extra dimensional coordinate. The matter field equation is obtained by variation of the action with respect to the tachyonic scalar field and it is expressed in the following form:

$$\partial_A \left[\frac{\sqrt{-g} V(T) \partial^A T}{\sqrt{1 + (\nabla T)^2}} \right] - \sqrt{-g} \sqrt{1 + (\nabla T)^2} \frac{\partial V(T)}{\partial T} = 0. \quad (4)$$

Thus, the braneworld field system consists of the bulk tachyonic scalar field, its self–interaction potential, and the metric functions which parameterize the line element (3).

The full solution for the metric scale and warp factors respectively reads

$$a(t) = e^{Ht}, \quad f(w) = \frac{1}{2} \ln [s \operatorname{sech} (H(2w + c))], \quad (5)$$

whereas the tachyon scalar field is given by

$$\begin{aligned} T(w) &= \pm \sqrt{\frac{-3}{2\kappa_5^2 \Lambda_5}} \operatorname{arctanh} \left(\frac{\sinh \left[\frac{H(2w+c)}{2} \right]}{\sqrt{\cosh [H(2w+c)]}} \right) \\ &= \pm \sqrt{\frac{-3}{2\kappa_5^2 \Lambda_5}} \ln \left[\tanh \left(\frac{H(2w+c)}{2} \right) + \sqrt{1 + \tanh^2 \left(\frac{H(2w+c)}{2} \right)} \right], \end{aligned} \quad (6)$$

while the tachyon potential has the form

$$\begin{aligned} V(T) &= -\Lambda_5 \operatorname{sech} \left(\sqrt{-\frac{2}{3}\kappa_5^2 \Lambda_5} T \right) \sqrt{6 \operatorname{sech}^2 \left(\sqrt{-\frac{2}{3}\kappa_5^2 \Lambda_5} T \right) - 1} \\ &= -\Lambda_5 \sqrt{(1 + \operatorname{sech} [H(2w+c)]) \left(1 + \frac{3}{2} \operatorname{sech} [H(2w+c)] \right)}, \end{aligned} \quad (7)$$

where H , c and $s > 0$ are arbitrary constants. We should mention that we have set

$$s = -\frac{6H^2}{\kappa_5^2 \Lambda_5}, \quad (8)$$

with a negative bulk cosmological constant $\Lambda_5 < 0$ when deriving the last two equations.

The corresponding curvature scalar reads

$$R = -\frac{14}{3}\kappa_5^2 \Lambda_5 \operatorname{sech} [H(2w+c)]. \quad (9)$$

As one can easily see, this invariant is positive definite (since the bulk cosmological constant is negative) and asymptotically flat along the fifth dimension. Thus, the thick braneworld under consideration interpolates between two 5D Minkowski spacetimes and is completely free of naked singularities that usually arise when the mass spectrum of Kaluza–Klein tensorial (graviton) fluctuations displays a mass gap, an appealing feature from the phenomenological viewpoint, as in this model (see [24] for details).

II. LOCALIZATION OF UNIVERSAL SCALAR FIELDS ON THE TACHYONIC DE SITTER BRANE

In this section we shall study the localization of bulk massive scalar fields on the above presented tachyonic de Sitter thick braneworld through gravitational interaction. It should be mentioned that we have implicitly assumed that the universal scalar fields considered below do make a small contribution to the energy of the bulk in such a way that the aforementioned solution to the field system remains valid in the presence of bulk scalar matter. Thus, the massive scalar fields we shall consider possess a quite small mass in order for this approximation to be valid. We shall further discuss the structure of the mass spectrum of the massive scalar field by analyzing the analog quantum mechanical potential of the corresponding Schrödinger equation for their KK modes and we shall present their general solution in terms of general Heun functions.

Thus, let us consider the action of a massive real scalar field minimally coupled to gravity

$$S = \int d^5x \sqrt{-g} \left(-\frac{1}{2} g^{MN} \partial_M \Phi \partial_N \Phi - \frac{m_s^2}{2} \Phi^2 \right), \quad (10)$$

where m_s is the bulk scalar field mass. With the aid of the metric (3), the equation of motion derived from (10) reads

$$\frac{1}{\sqrt{-\hat{g}}} \partial_\mu \left(\sqrt{-\hat{g}} \hat{g}^{\mu\nu} \partial_\nu \Phi \right) + e^{-3f} \partial_w (e^{3f} \partial_w \Phi) = 0. \quad (11)$$

We further propose the following ansatz for the KK decomposition of the scalar field $\Phi(x, w) = \sum_n \phi_n(x) \chi_n(w) e^{-3f/2}$ and demand that its 4D profile ϕ_n obeys a massive Klein–Gordon equation:

$$\left[\frac{1}{\sqrt{-\hat{g}}} \partial_\mu \left(\sqrt{-\hat{g}} \hat{g}^{\mu\nu} \partial_\nu \right) - m_n^2 \right] \phi_n(x) = 0, \quad (12)$$

where m_n is the 4D mass of the scalar field. On the other hand, the scalar KK mode $\chi_n(z)$ must satisfy a Schrödinger equation:

$$[-\partial_w^2 + V_s(w)] \chi_n(w) = m_n^2 \chi_n(w), \quad (13)$$

with the following effective quantum mechanical potential

$$V_s(w) = \frac{3}{2}f'' + \frac{9}{4}f'^2 + e^{2f}m_s^2. \quad (14)$$

It is evident that this potential only depends on bulk entities: the warp factor and the mass of the universal scalar field.

Thus, under the considered separation of variables, the 5D action (10) yields a standard 4D action for a massless and a series of massive scalar modes

$$S = -\frac{1}{2} \sum_n \int d^4x \sqrt{-\hat{g}} \left(\hat{g}^{\mu\nu} \partial_\mu \phi_n \partial_\nu \phi_n + m_n^2 \phi_n^2 \right), \quad (15)$$

after integrating along the fifth dimension, and requiring the fulfillment of both the following orthonormalization conditions

$$\int_{-\infty}^{\infty} \chi_m(w) \chi_n(w) dw = \delta_{mn} \quad (16)$$

and the Schrödinger equation (13).

Finally, for the warp factor solution (5) the analog quantum mechanical potential adopts the form

$$V_s = \frac{3H^2}{4} [3 - 7\text{sech}^2(2Hw)] + s m_s^2 \text{sech}(2Hw). \quad (17)$$

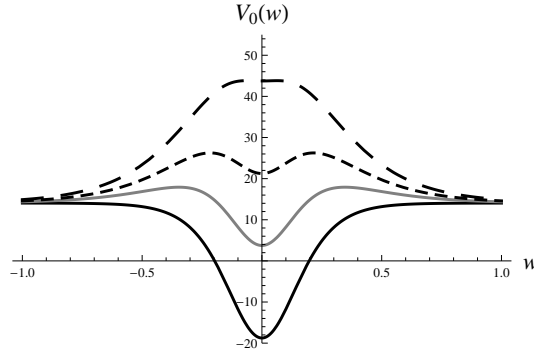


FIG. 1: The profile of the potential with $m_s = 0$ (thick line), $m_s = 3$ (thin line), $m_s = 4$ (dashed small lines), $m_s = 4$ (dashed large lines), along the fifth dimension. Here we have set $s = 2.5$, $H = 2.5$ and $k_1 = 1$ for simplicity.

In general, this potential can have a minimum or maximum at $w = 0$, and its behavior is governed by the dominant second or third term on the right hand side of (17) as depicted in Fig. 1. Generally, when $0 \leq m_s^2 < -\frac{7}{8}k_5^2\Lambda_5$ the dominant term is the second one and the potential is of modified Pöschl-Teller type with a massless KK state localized on the brane. However, as m_s^2 increases, the minimum of the Pöschl-Teller potential approaches a zero value and then becomes positive (namely, when $m_s^2 \geq -\frac{7}{8}k_5^2\Lambda_5$), leading to a Pöschl-Teller potential with no bound states which eventually becomes a potential barrier, making evident that there are not localized modes at all.

Thus, it is clear that in this case, the mechanism of universal scalar field localization depends not only on the geometry of bulk, but also on the bulk mass of the scalar field m_s , a parameter that plays a crucial role in the sense that it separates two localization phases: when it is small enough, leads to the localization of the massless zero mode on the brane, notwithstanding, when it increases and reaches a critical value, it delocalizes the full mass spectrum pushing all (massless and massive) KK modes outside the brane. Particularly, when $m_n = m_s = 0$ the potential well reaches its maximum depth and has a single localized bound state. We also can see that keeping the mass $m_n = 0$ and increasing the value of the mass in the bulk, the zero mode is slowly delocalized from the brane as m_s approaches the value $m_s^2 = -\frac{7}{8}k_5^2\Lambda_5$ (see Fig. 2).

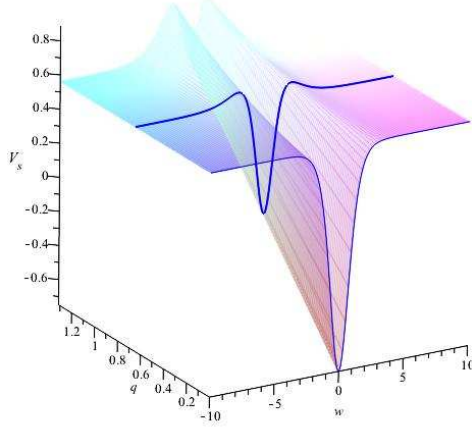


FIG. 2: The 3D surface shows the continuous shape of the potential for each point (x, q) with $x = 2Hw$, $q = \frac{3m_s^2}{2k_5^2 \Lambda_5}$ where the zero mode is located in the 2D domain surface $(-\infty, \infty) \times [0, \frac{21}{32})$, the first position where the zero KK mode is located in the potential is given by the thin blue line bounded by $m_s = 0$, and the least position where the zero KK mode is located in the potential is given by the thick blue line bounded by the critical value $m_s^2 = \frac{7k_5^2 \Lambda_5}{8}$.

Before presenting the solution corresponding to the KK mass spectrum problem on the equation (13) we will make the following change of variable $x = 2Hw$ then we shall the following anzats for the function $\chi(x)_n = \text{sech}^{\frac{3}{4}}(x)y(x)_n$, finally redefining the variable $x = \text{arcsech}(z)$ the equation (13) takes the following form

$$\frac{d^2 y(z)_n}{dz^2} + \frac{(7z^2 - 5)}{2z(z-1)(z+1)} \frac{dy(z)_n}{dz} + \frac{\left(\frac{3m_s^2}{2k_5^2 \Lambda_5} z - \frac{m_n^2}{4H^2}\right)}{z^2(z-1)(z+1)} y(z)_n = 0 \quad (18)$$

The general solution to the equation (18) with the reads

$$y(z)_n = \left[K_1 z^{-\frac{3H + \sqrt{9H^2 - 4m_n^2}}{4H}} (1-z)^{\frac{1}{2}} \text{HeunG} \left(-1, a_+, b_+, c_+, d_+, \frac{1}{2}, -z \right) + K_2 z^{-\frac{3H + \sqrt{9H^2 - 4m_n^2}}{4H}} (1-z)^{\frac{11}{4}} \text{HeunG} \left(-1, a_-, b_-, c_-, d_-, \frac{1}{2}, -z \right) \right], \quad (19)$$

where K_1 and K_2 are arbitrary constants, HeunG are general Heun special functions and the coefficients of the solution (19) are defined by

$$a_{\pm} = \frac{2Hk_5^2 \Lambda_5 \pm \sqrt{9H^2 - 4m_n^2}}{4Hk_5^2 \Lambda_5} + q, \quad b_{\pm} = \frac{9H \pm \sqrt{9H^2 - 4m_n^2}}{4H}, \quad c_{\pm} = -\frac{H \pm \sqrt{9H^2 - 4m_n^2}}{4H},$$

$$d_{\pm} = \frac{2H \pm \sqrt{9H^2 - 4m_n^2}}{2H}.$$

Then the whole solution has the form $\chi_n = \text{sech}^{\frac{3}{4}}(x)y(x)_n$.

In particular, for the massless zero mode bound state (with $m_n = 0$) starting from (18) we have the general Heun equation

$$\frac{d^2 y(z)_0}{dz^2} + \frac{(7z^2 - 5)}{2z(z-1)(z+1)} \frac{dy(z)_0}{dz} + \frac{q}{z(z-1)(z+1)} y(z)_0 = 0, \quad (20)$$

where $q = \frac{3m_s^2}{2k_5^2\Lambda_5}$. Now we can recast the zero mode solution solving the general Heung equation (20)

$$y(z)_0 = \left[C_1 z^{-\frac{3}{2}} \text{HeunG} \left(-1, q, -\frac{3}{2}, 1, -\frac{1}{2}, \frac{1}{2}, -z \right) + C_2 \text{HeunG} \left(-1, q, 0, \frac{5}{2}, \frac{5}{2}, \frac{1}{2}, -z \right) \right], \quad (21)$$

the complete solution for the zero mode in the x coordinate reads

$$\chi_0 = \text{sech}^{\frac{3}{4}}(x) \left[C_1 \cosh^{\frac{3}{2}}(x) \text{HeunG} \left(-1, q, -\frac{3}{2}, 1, -\frac{1}{2}, \frac{1}{2}, -\text{sech}(x) \right) + C_2 \text{HeunG} \left(-1, q, 0, \frac{5}{2}, \frac{5}{2}, \frac{1}{2}, -\text{sech}(x) \right) \right]. \quad (22)$$

However, only the second term corresponds to a localized configuration, therefore we need to set $C_1 = 0$. The corresponding extra dimensional profile of this zero mode is displayed in Fig. 3 within the range of allowed values for the bulk mass of the scalar field, i.e. for $0 \leq m_s^2 < -\frac{7}{8}k_5^2\Lambda_5$, which in turn implies the following bounds for the q parameter $0 \leq q < 21/32$.

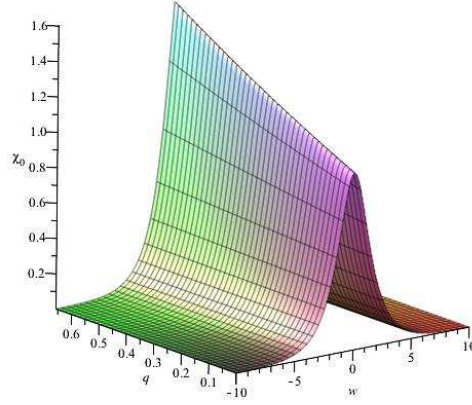


FIG. 3: This 3D figure shows the shape for the zero mode in the continuous range for $0 \leq q < \frac{21}{32}$ which corresponds to the critical values $m_s = 0$ and $m_s^2 = \frac{7k_5^2\Lambda_5}{8}$.

Within the framework of a little bit less general case we can consider that $m_s \ll 1$ in order to guarantee the localization of the only discrete state of the KK mass spectrum on the brane. Thus, by neglecting the small parameter m_s , our potential (17) adopts the approximate form

$$V_s = \frac{3H^2}{4} [3 - 7\text{sech}^2(2Hw)]. \quad (23)$$

This approximated potential still has a mass gap given by $\frac{9H^2}{4}$ in the spectrum of KK massive fluctuations of the scalar field. By redefining the extra coordinate $u = 2Hw$ the Schrödinger equation (13) transforms into [22]

$$\left(-\partial_u^2 - \frac{21}{16}\text{sech}^2 u \right) \chi(u) = \left(\frac{m_n^2}{4H^2} - \frac{9}{16} \right) \chi(u), \quad (24)$$

and adopts the canonical form of the classical eigenvalue problem for the Schrödinger equation with a quantum mechanical potential of modified Pöschl–Teller type:

$$[-\partial_u^2 - n(n+1)\text{sech}^2 u] \chi(u) = E \chi(u) \quad (25)$$

where $n = 3/4$ and now $E = \frac{m_s^2}{4H^2} - \frac{9}{16}$. Since $n < 1$, then $[n] = 0$ and, as it is already known, there is just one bound state, the massless one, in the mass spectrum of the KK fluctuations of the universal scalar field. It turns out that (24) can be integrated for any arbitrary mass and possess the following general solution:

$$\chi(w) = c_1 P_{3/4}^\mu(\tanh(2Hw)) + c_2 Q_{3/4}^\mu(\tanh(2Hw)) \quad (26)$$

where c_1, c_2 are integration constants, and $P_{3/4}^\mu$ and $Q_{3/4}^\mu$ are associated Legendre functions of first and second kind, respectively, with degree $\nu = 3/4$ and order $\mu = \sqrt{\frac{9}{16} - \frac{m_s^2}{4H^2}}$. It turns out that for the massless zero mode $\mu = \nu = 3/4$ and the exact solution can be written as follows

$$\chi_0(w) = -\alpha_1 \left[P_{3/4}^{3/4}(\tanh(2Hw)) + \frac{2}{\pi} Q_{3/4}^{3/4}(\tanh(2Hw)) \right], \quad (27)$$

where $c_1 = -\alpha_1$, $\alpha_1 > 0$ and we have chosen $c_2 = 2c_1/\pi$ in order to obtain a localized field configuration along the fifth dimension. This massless bound state corresponds to the localized zero mode of the scalar field on the 3-brane. The profile of both the modified Pöschl–Teller potential and the massless zero mode of the scalar field along the extra dimension is displayed in Fig. 4.

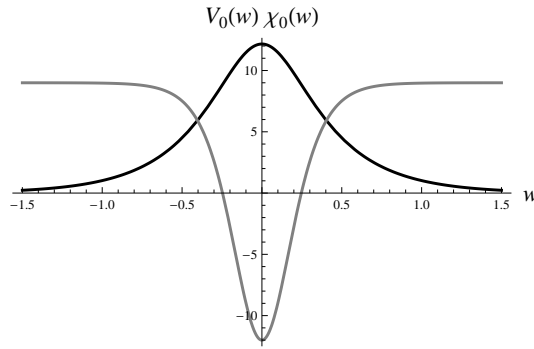


FIG. 4: The profile of the modified Pöschl–Teller potential (thin line) and the localized zero mode (thick line) along the fifth dimension. Here we have set $H = 2$ and $\alpha_1 = 10$ for simplicity.

III. CONCLUSIONS AND DISCUSSION

Extra dimensional models like braneworlds are of interest to look for new physics not only because of the possible deviations from standard physics they may lead to, but because they may inspire further, perhaps more precise tests, not previously noticed; this was the case, for instance, for the experiments proving Newton's law at the submillimeter length scale. In order to elucidate any new effect, one has to contrast it with 4D data and/or observations and typically this emerges within the model through a localization mechanism for matter and gravity.

In this work we have investigated the localization properties of massive universal scalar field in a tachyonic de Sitter thick braneworld which interpolates between two Minkowski 5D spacetimes. The dynamics of the scalar field is governed by a Schrödinger equation with an analog quantum mechanical potential completely determined by the 5D warping and the 5D mass of the free scalar field. We have found that this bulk mass plays the role of a continuous parameter which can delocalize all the mass spectrum of the scalar field, i.e. the massless bound state and the massive KK ones, from the expanding 3-brane when it reaches a critical value. Thus, we have uncovered two localization phases of the 4D massless zero mode depending on how small is the mass of the bulk scalar field compared to a product of the 5D cosmological and gravitational constants. In fact, a result of this nature is to be expected due to the assumption we made that the 5D scalar field does not interact with the gravitational field in the bulk leaving the background solution valid, and neglecting its gravitational back-reaction.

On the basis of the performed analysis of the structure of the analog quantum mechanical potential one can easily see how sensitive is our model, where the coupling constant m_s can drag the 4D massless zero mode of the scalar field outside the brane. Thus, the matter density on the bulk must be located far away from the brane in order to make a small contribution to the parameter m_s in such a way that we could have the KK massless zero mode localized inside the brane. It is worth noticing that, usually, when one takes into account the gravitational back-reaction, the

scalar modes (both massless and massive) are not localized on the brane anymore, in accordance with the previously described result since the 5D mass of the scalar field can be interpreted as a measure of its gravitational back-reaction.

A similar and interesting situation arises within the context of the AdS/CFT correspondence: when computing the amplitudes of a scalar field, the effect of its interaction with its dual operator depends on the quotient $\left(\frac{\text{energy}}{\text{mass}}\right)^\alpha$, for some constant $\alpha > 0$. Thus, three different situations arise, depending on the value of the scalar field mass m^2 , which in an AdS space is allowed to acquire negative values as far as the so-called Breitenlohner-Freedman bound is satisfied. It turns out that for $m^2 > 0$ the effects of the interaction of the scalar field with its dual operator, called irrelevant in this case, can be neglected for low energies since they are suppressed by powers. Therefore, the interaction goes away in the IR, but changes completely the UV sector of the theory. When $m = 0$ the corresponding dual operator is called marginal. Finally, if m^2 adopts the allowed negative values, the dual operator is called relevant and changes the IR sector of the theory (see, for instance, [41] for a clear exposition of this issue).

Thus, along with the gravitational and gauge vector fields, the 4D scalar fields can be localized on the considered tachyonic de Sitter thick braneworld, if the mass of the bulk scalar field is bounded from above. It remains to determine whether fermions can be localized on this kind of braneworld configurations, however, one should first overcome the technical difficulties that this models presents in such a study. In the case of fermion localization on the tachyonic expanding 3-brane, one would be able to compute the corrections to Coulomb law coming from the KK massive modes of the gauge vector field in the non-relativistic limit. Since the photon mass is very suppressed, this study could lead, in principle, to establish whether this braneworld model is viable or not from the phenomenological point of view.

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